1. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	I	II	III
I	5	2	3
II	3	5	4

(a) Verify that there is no stable solution to this game.

(3)

(b) Find the best strategy for player *A* and the value of the game to her.

(8)
(Total 11 marks)

2. A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–4	5	1
A plays 2	3	–1	–2
A plays 3	–3	0	2

Formulate the game as a linear programming problem for player A. Write the constraints as inequalities and define your variables.

(Total 7 marks)

3. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–5	6	–3
A plays 2	1	– 4	13
A plays 3	–2	3	–1

(a) Verify that there is no stable solution to this game.

(3)

(b) Reduce the game so that player B has a choice of only two actions.

(1)

(c) Write down the reduced pay-off matrix **for player B**.

(2)

(d) Find the best strategy for player B and the value of the game to player B.

(7)
(Total 13 marks)

4. Laura (L) and Sam (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Laura.

	S plays 1	S plays 2	S plays 3
L plays 1	–2	8	–1
L plays 2	7	4	–3
L plays 3	1	–5	4

Formulate the game as a linear programming problem for Laura, writing the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

5. (a) In game theory, explain the circumstances under which column (x) dominates column (y) in a two-person zero-sum game.

(2)

Liz and Mark play a zero-sum game. This game is represented by the following pay-off matrix for Liz.

	Mark plays 1	Mark plays 2	Mark plays 3
Liz plays 1	5	3	2
Liz plays 2	4	5	6
Liz plays 3	6	4	3

(b) Verify that there is no stable solution to this game.

(3)

(c) Find the best strategy for Liz and the value of the game to her.

(9)

The game now changes so that when Liz plays 1 and Mark plays 3 the pay-off to Liz changes from 2 to 4. All other pay-offs for this zero-sum game remain the same.

(d) Explain why a graphical approach is no longer possible and briefly describe the method Liz should use to determine her best strategy.

(2)
(Total 16 marks)

6. Denis (D) and Hilary (H) play a two-person zero-sum game represented by the following pay-off matrix for Denis.

	H plays 1	H plays 2	H plays 3
D plays 1	2	–1	3
D plays 2	–3	4	–4

(a) Show that there is no stable solution to this game.

(3)

(b) Find the best strategy for Denis and the value of the game to him.

(10)
(Total 13 marks)

7. Anna (A) and Roland (R) play a two-person zero-sum game which is represented by the following pay-off matrix for Anna.

	R plays 1	R plays 2	R plays 3
A plays 1	6	–2	–3
A plays 2	–3	1	2
A plays 3	5	4	–1

Formulate the game as a linear programming problem **for player R**. Write the constraints as inequalities. Define your variables clearly.

(Total 8 marks)

8. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	5	7	2
A plays 2	3	8	4
A plays 3	6	4	9

(a) Formulate the game as a linear programming problem for player *A*, writing the constraints as equalities and clearly defining your variables.

(5)

(b) Explain why it is necessary to use the simplex algorithm to solve this game theory problem.

(1)

(c) Write down an initial simplex tableau making your variables clear.

(2)

(d) Perform two complete iterations of the simplex algorithm, indicating your pivots and stating the row operations that you use.

(8)
(Total 16 marks)

9. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	–2	1	3	–1
A plays 2	–1	3	2	1
A plays 3	–4	2	0	–1
A plays 4	1	–2	–1	3

(a) Verify that there is no stable solution to this game.

(3)

(b) Explain why the 4 × 4 game above may be reduced to the following 3 × 3 game.

–2	1	3
–1	3	2
1	–2	–1

(2)

(c) Formulate the 3 × 3 game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly.

(8)
(Total 13 marks)

10. (a) Explain briefly what is meant by a **zero-sum** game.

(1)

A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	I	II	III
I	5	2	3
II	3	5	4

(b) Verify that there is no stable solution to this game.

(3)

(c) Find the best strategy for player *A* and the value of the game to her.

(8)

(d) Formulate the game as a linear programming problem for player *B*. Write the constraints as inequalities and define your variables clearly.

(5)
(Total 17 marks)

11. In game theory explain what is meant by

(a) zero-sum game,

(2)

(b) saddle point.

(2)
(Total 4 marks)

12. Emma and Freddie play a zero-sum game. This game is represented by the following pay-off matrix for Emma.

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$$

(a) Show that there is no stable solution.

(3)

(b) Find the best strategy for Emma and the value of the game to her.

(8)

(c) Write down the value of the game to Freddie and his pay-off matrix.

(3)
(Total 14 marks)

13. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	–3	2	5
A plays II	4	–1	–4

(a) Write down the pay off matrix for player *B*.

(2)

(b) Formulate the game as a linear programming problem for player *B*, writing the constraints as equalities and stating your variables clearly.

(4)
(Total 6 marks)

14. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0
A plays III	0	1	–3

(a) Identify the play safe strategies for each player.

(4)

(b) Verify that there is no stable solution to this game.

(1)

(c) Explain why the pay-off matrix above may be reduced to

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0

(2)

(d) Find the best strategy for player *A*, and the value of the game.

(7)
(Total 14 marks)

15. Members of a string quartet have been playing together for 20 years. At their concerts they sometimes play mostly "old music", sometimes they play mainly "new music" and sometimes they play a mixture of both old and new. The choice of music depends on the age of the audience, provided this is known in advance. Young audiences prefer new music and older audiences prefer old music.

The table shows the audience reaction, as a score out of 10, for each of the possible combinations.

		Audience		
		Young	**Mixture**	**Old**
Music	**Old**	1	3	8
	Mixture	4	2	3
	New	7	5	3

(a) Explain why the quartet should never choose to play a mixture of old and new music.

(2)

Generally, the quartet does not know in advance whether their audience will be mostly young, mostly old or a mixture.

If they choose to play old music with a probability p and new music with a probability $(1 - p)$,

(b) calculate the expected reaction, as a score out of 10, from each of the three types of audience.

(3)

(c) Use a graphical method to decide what value p should take to maximise the minimum expected reaction from part (*b*). Explain your method carefully.

(9)
(Total 14 marks)

1. (a)

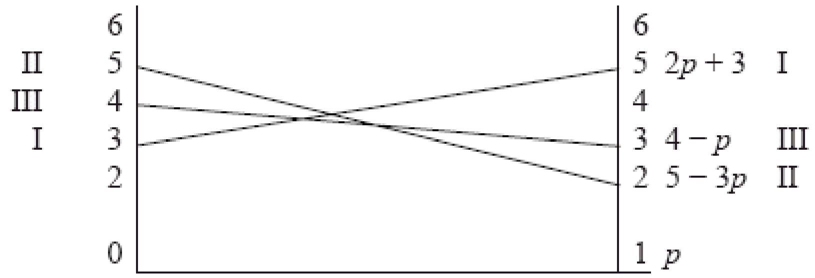
	I	II	III	
I	5	2	3	Min 2
II	3	5	4	Min 3 ← max
	Max 5	5	4	
			min	

M1A1

Since 3 $\neq$ 4 not stable — A1 — 3

(b) Let *A* play I with probability *p*
Let *A* play II with probability (1 – *p*)
If *B* plays I *A*'s gain are $5p + 3(1 - p) = 2p + 3$
If *B* plays II *A*'s gain are $2p + 5(1 - p) = 5 - 3p$ — M1 A1 — 2

If *B* plays III *A*'s gain are $3p + 4(1 - p) = 4 - p$



A 2, 1, 0 — 2

Intersection of $2p + 3$ and $4 - p$ $\quad p = \frac{1}{3}$ — M1 A1ft — 2

A should play I of time and II of time; value (to *A*) = 3 — A1 ft A1 ft — 2

[11]

2. $\begin{bmatrix} -4 & 5 & 1 \\ 3 & -1 & -2 \\ -3 & 0 & 2 \end{bmatrix} \rightarrow$ add 5 to all entries $\begin{bmatrix} 1 & 10 & 6 \\ 8 & 4 & 3 \\ 2 & 5 & 7 \end{bmatrix}$ — M1

Either	**Or**	
Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively.	Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively.	B1
	Let $x_i = \frac{p_i}{V}$	
Maximise P = V	Minimise P $= x_1 + x_2 + x_3$	B1
Subject to:	Subject to	
$V - p_1 - 8p_2 - 2p_3 \le 0$	$x_1 + 8x_2 + 2x_3 \ge 1$	M1
$V - 10p_1 - 4p_2 - 5p_3 \le 0$	$10x_1 + 4x_2 + 5x_3 \ge 1$	A1
$V - 6p_1 - 3p_2 - 7p_3 \le 0$	$6x_1 + 3x_2 + 7x_3 \ge 1$	A1
$p_1 + p_2 + p_3 \le 1$	$x_1, x_2, x_3 \ge 0$	A1

$p_1, p_2, p_3 \geq 0$

–

Note

1M1: Adding n (≥ 4) to all entries
1B1: Defining variables
1B1: Objective correct
2M1: At least 3 constraints, using columns, one of correct form
1A1ft: one correct constraint – excluding non-negativity constraint
2A1ft: two correct constraints – excluding non-negativity constraint
3A1: cao including non-negativity constraint

[7]

3. (a) Row minima {–5, –4, –2} row maximin = –2

Column maxima {1, 6, 13} col minimax = 1M1 A1

–2 ≠ 1 therefore not stable. A1 3

(b) Column 1 dominates column 3, so column 3 can be deleted. B1 1

(c)

	A plays 1	A plays 2	A plays 3
B plays 1	5	–1	2
B plays 2	–6	4	–3

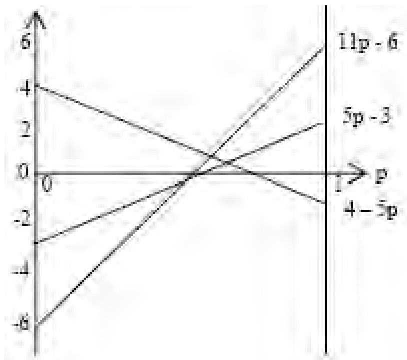
B1 B1 2

(d) Let B play row 1 with probability p and row 2 with probability (1-p)

If A plays 1, B's expected winnings are 11p – 6 M1 A1

If A plays 2, B's expected winnings are 4 – 5p

If A plays 3, B's expected winnings are 5p – 3



M1 A1

$5p - 3 = 4 - 5p$ M1

$10p = 7$

$p = \frac{7}{10}$

B should play 1 with a probability of 0.7

2 with a probability of 0.3 A1

and never play 3

The value of the game is 0.5 to B A1 7

[13]

4. E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$ B1

Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively

Let V = value of game + 6 B1

e.g.

Maximise P = V

Subject to:

$V - 4p_1 - 13p_2 - 7p_3 \leq 0$ M1

$V - 14p_1 - 10p_2 - p_3 \leq 0$ A3 2ft 1ft 0

$V - 5p_1 - 3p_2 - 10p_3 \leq 0$

$p_1 + p_2 + p_3 \leq 1$

$p_1, p_2, p_3 \geq 0$

Note

1B1: Making all elements positive

2B1: Defining variables

3B1: Objective, cao word and function

1M1: At least one constraint in terms of their variables, must be going down columns.

Accept = here.

1A1ft: ft their table. One constraint in V correct.

2A1ft: ft their table. Two constraints in V correct.

3A1: CAO all correct.

Alt using x_i method

Now additionally need: let $x_i = \frac{Pi}{v}$ for 2B1

minimise (P) = $x_1 + x_2 + x_3 = \frac{1}{v}$

subject to:

$4x_1 + 13x_2 + 7x_3 \geq 1$

$14x_1 + 10x_2 + x_3 \geq 1$

$5x_1 + 3x_2 + 10x_3 \geq 1$

$x_i \geq 0$

[7]

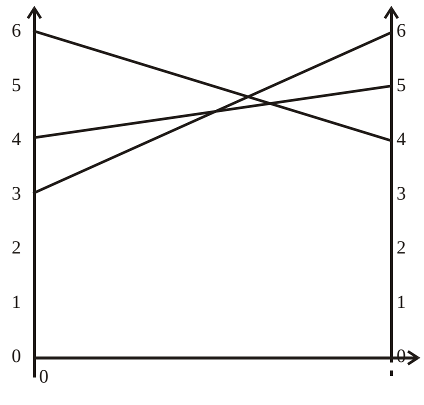
5. (a) For each row the element in column x must be less than the element in column y. B2,1,01 2

(b) Row minimum {2,4,3} row maximin = 4 M1
Column maximum {6,5,6} column minimax = 5 A1
4 ≠ 5 so not stable A1 3

(c) Row 3 dominates row 1, so matrix reduces to B1

	M1	**M2**	**M3**
L2	4	5	6
L3	6	4	3

Let Liz play 2 with probability p and 3 with probability (1– p)
If Mark plays 1: Liz's gain is 4p + 6 (1– p) = 6 – 2p
If Mark plays 2: Liz's gain is 5p + 4 (1– p) = 4 + p M1
If Mark plays 3: Liz's gain is 6p + 3 (1– p) = 3 + 3p A1 3



B2,1,0 2

$4 + p = 6 - 2p$ M1A1

$p = \frac{2}{3}$ A1ftA1 4

(d) Liz should play row 1 – never, row 2 – $\frac{2}{3}$ of the time,

row 3 – $\frac{1}{3}$ of the time

and the value of the game is $4\frac{2}{3}$ to her. B1

Row 3 no longer dominates row 1 and so row 1 can not be deleted.
Use Simplex (linear programming). B1 2

[16]

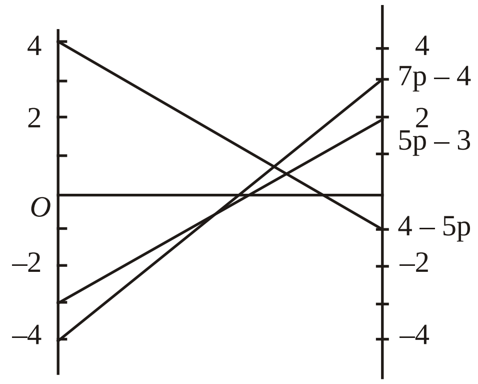
6. (a)

$$\begin{bmatrix} 2 & -1 & 3 \\ -3 & 4 & -4 \end{bmatrix} \quad \begin{matrix} \text{Row min} \\ -1 \leftarrow \\ -4 \end{matrix}$$ M1A1

col max 2 ↑ 4 3 $2 \neq -1 \therefore$ not stable A1 3

(b) Let Denis play 1 with probability p
So he'll play 2 with probability $1 - p$

If Hilary plays 1 Denis wins: $2p - 3(1 - p) = 5p - 3$ M1
If Hilary plays 2 Denis wins: $-p + 4(1 - p) = 4 - 5p$ A2,1,0
If Hilary plays 3 Denis wins: $3p - 4(1 - p) = 7p - 4$



M1A2,1,0

$5p - 3 = 4 - 5p$
$10p = 7$
$p = \frac{7}{10}$ M1A1ft

Denis should play 1 with probability $\frac{7}{10}$

2 with probability $\frac{3}{10}$

the value of the game is $\frac{1}{2}$ B1ftB1 10

[13]

7. Alt 1

Game from R's point of view.

	A1	A2	A3
R_1	–6	3	–5
R_2	2	–1	–4
R_3	3	–2	1

Add 7

	A1	A2	A3
R_1	1	10	2
R_2	9	6	3
R_3	10	5	8

B1, B1

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 B1
V = value of the game

Maximise P = V B1

Subject to $V - P_1 - 9P_2 - 10P_3 \le 0$ M1A1ft
$V - 10P_1 - 6P_2 - 5P_3 \le 0$ A1ft
$V - 2P_1 - 3P_2 - 8P_3 \le 0$ A1ft
$P_1 + P_2 + P_3 \le 1$ accept= A1 8
$V, P_1, P_2, P_3 \ge 0$

Alt 2

Add 4 to all entries B1

	R_1	R_2	R_3
A1	10	2	1
A2	1	5	6
A3	9	8	3

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 let V = value of game. B1

Let $x_1 = \frac{P_1}{V}, x_2 = \frac{P_2}{V}, x_3 = \frac{P_3}{V}$ B1

Maximise $P = x_1 + x_2 + x_3$ B1

Subject to $10x_1 + 2x_2 + x_3 \le 1$ M1A1ft
$x_1 + 5x_2 + 6x_3 \le 1$ A1ft
$9x_1 + 8x_2 + 3x_3 \le 1$ A1
$x_1, x_2, x_3 \ge 0$ accept $P_i \ge 0$

[8]

8. (a) e.g. Maximise P = V — B1

Subject to: $V - 5p_1 - 3p_2 - 6p_3 + r = 0$ — M1

$V - 7p_1 - 8p_2 - 4p_3 + s = 0$ — A2,1,0

$V - 2p_1 - 4p_2 - 9p_3 + t = 0$

$p_1 + p_2 + p_3 (+ u) = 1$

where V = value of game to A, P_i = probability of A playing row i

$P_i \geq 0$ and r, s, t, u are stack variables all $\geq$ 0 — B1 — 5

B1 Maximise/minimise and consistent function
M1 constraints (condone non-negativity)
– at least one correct <u>must be equations</u>
A2 all correct
A1 at least two correct
B1 defining variables

(b) Not reducible and a three variable problem — B1 — 1

B1 cao – both

(c) e.g.

b v	V	P_1	P_2	P_3	r	s	t	u	value		
r	**1**	–5	–3	–6	1	0	0	0	0	M1	
s	1	–7	–8	–4	0	1	0	0	0	A1	
t	1	–2	–4	–9	0	0	1	0	0		2
u	0	1	1	1	0	0	0	1	1		
P	–1	0	0	0	0	0	0	0	0		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops		
V	1	–5	–3	–6	1	0	0	0	0	$R_1 / 1$	M1 A1	
s	0	–2	–5	**2**	–1	1	0	0	0	$R_2 - R_1$	A1	
t	0	–3	–1	–3	–1	0	1	0	0	$R_3 - R_1$	B1ft	
u	0	1	1	1	0	0	0	1	1	R_4 stet		4
P	0	–5	–3	–6	1	0	0	0	0	$R_5 + R_1$		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops	
V	1	–11	–18	0	–2	3	0	0	0	$R_1 + 6R_2$	M1 A1ft
P_3	0	–1	$-\frac{5}{2}$	1	½	½	0	0	0	$R_2 / 2$	A1
t	0	0	$-\frac{17}{2}$	0	$-\frac{5}{2}$	$\frac{5}{2}$	1	0	0	$R_3 + 3R_2$	B1ft

u	0	2	$\frac{7}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	1	$R_4 - R_2$	4
P	0	–11	–18	0	–2	3	0	0	0	$R_5 + 6R_2$	

[16]

9. (a) Row minimums {–2, –1, –4, –2} row maximum = –1 M1
Column maximums {1, 3, 3, 3} column minimum = 1 A1

Since 1 ≠ –1 not stable A1 3

(b) Row 2 dominates Row 3 B1
Column 1 dominates column 4 B1 2

(c) Let A play row R, with probability P_1, R_2 with probability P_2 and "R_3" with probability P_3. B1

$$\begin{pmatrix} -2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{pmatrix} \begin{matrix} \text{eg} \\ \rightarrow \\ +3 \end{matrix} \begin{pmatrix} 1 & 4 & 6 \\ 2 & 6 & 5 \\ 4 & 1 & 2 \end{pmatrix}$$ M1 2

e.g. maximise P = V M1 A1

subject to $V - p_1 - 2p_2 - 4p_3 \leq 0$ A4ft, 3ft, 2ft, 1ft, 0 6

$V - 4p_1 - 6p_2 - p_3 \leq 0$

$V - 6p_1 - 5p_2 - 2p_3 \leq 0$

$p_1 + p_2 - p_3 \leq 1$

$V, p_1, p_2, p_3 \geq 0$

OR

e.g. Let $x_i = \frac{p_i}{v}$ $\therefore$ $\frac{1}{v} = x_1 + x_2 + x_3$ M1

minimise $P = x_1 + x_2 + x_3$ A1

subject to $x_1 + 2x_2 + 4x_3 \geq 1$
$4x_1 + 6x_2 + x_3 \geq 1$ A4ft 3ft 2ft 1ft 0 6
$6x_1 + 5x_1 + 2x_3 \geq 1$
$x_1 + x_2 + x_3 \geq 0$

+ other equivalent methods.

[13]

10. (a) A zero-sum game is one in which the sum of the gains for all players is zero. (o.e.) B1 1

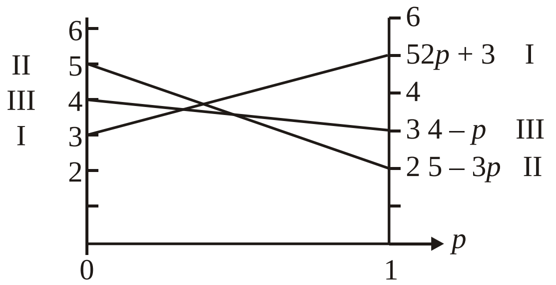
(b) I II III

I	5	2	3	min 2	
II	3	5	4	min 3 ← max	M1 A1
	max 5	5	4		
			↑ min		

Since 3 ≠ 4 not stable A1 3

(c) Let A play I with probability p
Let A play II with probability $(1 – p)$

If B plays I A's gains are $5p + 3(1 – p) = 2p + 3$
If B plays II A's gains are $2p + 5(1 – p) = 5 – 3p$ M1 A1 2
If B plays III A's gains are $3p + 4(1 – p) = 4 – p$



A2,1,0 2

Intersection of $2p + 3$ and $4 – p \Rightarrow p = \frac{1}{3}$ M1 A1ft 2

$\therefore$ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) = $3\frac{2}{3}$ A1ft A1ft 2

(d) Let B play I with probability q_1,
II with probability q_2 and
III with probability q_3 B1

e.g. Let $x_1 = \frac{q_1}{v}$ $x_2 = \frac{q_2}{v}$ $x_3 = \frac{q_3}{v}$ M1

Maximise $P = x_1 + x_2 = x_3$ A1
subject to $5x_1 + 2x_2 + 3x_3 \le 1$
$3x_1 + 5x_2 + 4x_3 \le 1$ A2,1,0 5
$x_1, x_2, x_3 \ge 0$

[17]

Alt 1

e.g. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$

maximise P = V
subject to $v – q_1 – 4q_2 – 3q_3 \le 0$
$v – 3q_1 – q_2 – 2q_3 \le 0$ $q_1 + q_2 + q_3 \le 1$
$v, q_1, q_2, q_3 \ge 0$ or = 1

11. (a) A game in which the gain to one player is equal to the loss of the other — B2, 1, 0 — 2

(b) If there is a <u>stable solution</u>(s) a_{ij} in a game, the <u>location</u> of this stable solution is called the saddle point. — B2, 1, 0 — 2
It is the point(s) where row maximum = column maximum.

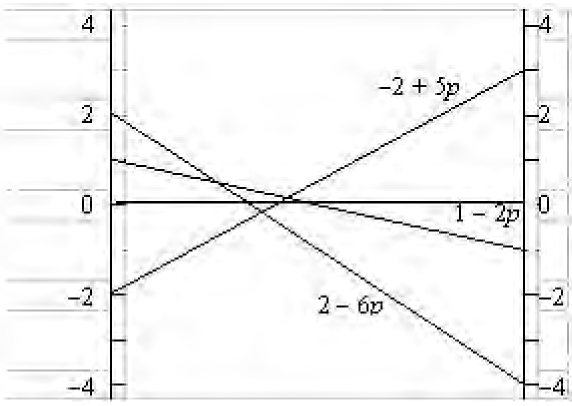
[4]

12. (a)

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix} \quad \begin{matrix} \text{row min} \\ -4 \\ -2 \leftarrow \text{max} \end{matrix}$$

Col. max 2 1 3 — M1 A1
↑
min
$-2 \neq 1 \therefore$ not stable — A1 — 3

(b) Let Emma play R_1 with probability p
If Freddie plays C_1, Emma's winnings are $-4p + 2(1 - p) = 2 - 6p$
C_2, Emmas winnings are $-p + 1(1 - p) = 1 - 2p$ — M1 A1
C_3, Emma's winnings are $3p - 2(1 - p) = -2 + 5p$ — A1 — 3



M1 A1 ft — 2

Need intersection of $2 - 6p$ and $-2 + 5p$ — M1
$2 - 6p = -2 + 5p,$
$4 = 11p,$
$p = \frac{4}{11}$ — A1

So Emma should play R_1 with probability $\frac{4}{11}$
R_2 with probability $\frac{7}{11}$ — A1 ft — 3

The value of the game is $-\frac{2}{11}$ to Emma

(c) Value to Freddie $\frac{2}{11}$, matrix $\begin{pmatrix} 4 & -2 \\ 1 & -1 \\ -3 & 2 \end{pmatrix}$ B1 ft B1, B1 3

[14]

13. (a)

	A(I)	A(II)
B(I)	3	–4
B(II)	–2	1
B(III)	–5	4

B2, 1, 0 2

(b) e.g. matrix becomes

	A(I)	A(II)
B(I)	9	2
B(II)	4	7
B(III)	1	10

M1

Defines variables (–including non-zero constants) B1

e.g. maximise $P = V$

subject to $v - 9q_1 - 4q_2 - q_3 + r = 0$

$v - 2q_1 - 7q_2 - 10q_3 + s = 0$

$q_1 + q_2 + q_3 + t = 1$

OR

e.g. minimise $P = x_1 + x_2 + x_3$ where $x_i = \frac{q_i}{v}$

subject to $9x_1 + 4x_2 - x_3 + r = 1$

$2x_1 - 7x_2 - 10x_3 + s = 1$ A2 ft, 1 ft, 0 4

OR

e.g. maximise $P = V$

$v - 8q_1 - 3q_2 + R = 0$

$v - 8q_1 - 3q_2 + S = 0$

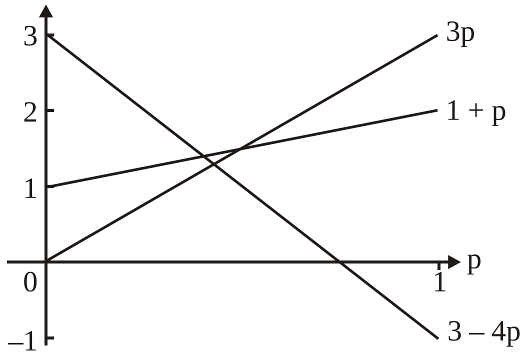
[6]

14. (a) Player A: row minimums are –1, 0, –3 so maximin choice is play II M1 A1

Player B: column maximums are 2, 3, 3 so minimax choice is play I M1 A1 4

(b) Since A's maximin (0) ≠ B's minimax (2) there is no stable solution B1 1

(c)	For player A row II dominates row III, so A will *now* play III	B2, 1, 0	2
(d)	Let A play I with probability p and II with probability $(1 - p)$ If B plays I, A's expected winnings are $2p + (1 - p) = 1 + p$ If B plays II, A's expected winnings are $-p + 3(1 - p) = 3 - 4p$ If B plays III, A's expected winnings are $3p$	M1, A2, 1, 0	3



M1

$3 - 4p = 3p \Rightarrow \underline{p = \frac{3}{7}}$	A1	
A should play I with probability $\frac{3}{7}$		
A should play II with probability $\frac{4}{7}$	A1	
and never play III		
The value of the game is $\underline{\frac{9}{7}\text{ to } A}$	A1 ft	4

[14]

15. (a) Each entry in row 2 is less than or equal to the corresponding entry in row 3,

$(4 < 7, 2 < 5, 3 \leq 3)$ — B1

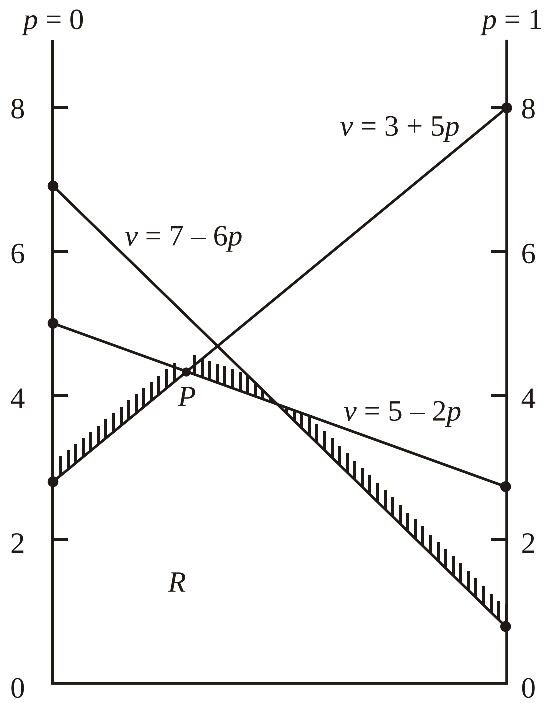
They may therefore always do better, or no worse, by choosing “new” rather than “mixed” — B1 — 2

(b)

Young: expected reaction	$1p + 7(1 - p) = 7 - 6p$		
Mixed: expected reaction	$3p + 5(1 - p) = 5 - 2p$	M1 A1 A1	3
Old: expected reaction	$8p + 3(1 - p) = 3 + 5p$		

(c) Let v be the expected reaction (value). Then the quartet wish to maximise v subject to:

$v \le 7 - 6p$, $v \le 5 - 2p$, $v \le 3 + 5p$ by appropriate choice of p M1 A1



The inequalities are satisfied in the region R. The maximum value of v occurs at P where

$3 + 5p = 5 - 2p$ M1 A1

So $7p = 2$, $p = \frac{2}{7}$ and $v = 3 + 5p = 3 + \frac{10}{7} = 4\frac{3}{7}$ M1 A1 9

[14]

1. No Report available for this question.

2. Candidates commonly either obtained very few marks for this question, or virtually full marks. Some did not attempt the question. Some attempted to solve the problem instead of formulating it as a linear programming problem. Many failed to add 5 to the elements and/or define their probabilities and/or state that they were maximising $P = v$. Many attempts involved 9 variables. Many used the rows of the table rather than the columns and some used slack variables and equalities rather than the inequalities requested. Some omitted the non-negativity constraints. Despite this a good number, nearly 20% were able to get full marks on this question.

3. This also proved a good discriminator. Part (a) was often a good source of marks for most. In part (b) most candidates deleted column 3, although a significant number deleted column 1. The matrix transposition and/or sign changes were frequently omitted in part (c). Most candidates were able to set up three probability expressions with relatively few algebraic errors, but the graphs were often poorly drawn, some without a ruler or linear scale. A significant number of candidates were not able to correctly select the optimal point.

4. This proved to be the most challenging question on the paper, with many attempts marking little progress, but very few blanks seen.. Many candidates did not answer the question as stated and did not give their answer from Laura's point of view and did not give their constraints as inequalities. Very few added 6 to all terms to make the entries positive. Most did attempt to define their variables, but some tried to structure the problem in terms of nine unknowns, confusing Game Theory with Transportation or Allocation. Many stated an objective but then either did not state if they were maximising or minimising or selected the wrong one. The majority of candidates used rows instead of columns to set up the constraints and those that sis correctly use columns often made mistakes with the inequality signs. Very many candidates unnecessarily made two attempts at the question, firstly in terms of V and p's and then in terms of x.

5. Most candidates were able to give a reasonable definition of dominance, although some confused column and row dominance. Many made rather sweeping statements such as 'the values in the x column are less than the values in the y column', rather than preceding this by 'for each row'. The majority of candidates answered part (b) well, but some found the row maximin and column minimax or did not identify the two key values from their row minimums and column maximums. Algebraic errors were pleasing rare in part (c) this year and most were able to find the correct three equations and draw them accurately. Examiners are still disappointed by the number of candidates who do not use a ruler to draw straight lines, and a number of candidates continued their lines beyond the domain for a probability. The optimal point was less regularly correctly identified with many choosing the intersection of the lines 3 + 3p and 6 – 2p and giving a popular incorrect solution of 3/5. A significant number of candidates did not state the probability for each of Liz's <u>three</u> options and/or the value of the game. Part (d) was usually well answered, but a few wasted time setting up a simplex tableau.

6. This question was well done by the majority of candidates. Most completed (a) well, but a few found the wrong values for row maximin and column minimax, many did not make clear the link to show that the game was not stable. In part (b), most candidates clearly defined their probabilities and set up the three equations correctly, although there were then some errors when simplifying these. Many errors occurred when drawing the graph, including a lack of a scale, labels, ruler (!) and going outside the (probability) domain, as well plotting lines incorrectly. Some candidates did not draw a graph at all. A significant number of candidates chose the wrong intersection point to calculate the value of p. Most candidates went on to state their strategy and value of the game.

7. This question was poorly done by a large number of candidates. There are two common approaches to this question. The majority of candidates used the ‘dividing by V’ method (alt 2 on the mark scheme). Common errors included failing to add 4 to the matrix, not defining their probabilities and not realising that they should be maximizing 1/V. When setting up the inequalities, some candidates used columns instead of rows, some did not fully divide by V and some reversed the inequalities or incorrectly introduced slack variables. Some candidates also forgot the non-negativity constraints. Those candidates who chose the ‘transposing’ method (alt 1 on the mark scheme) generally produced a better attempt, although a significant number did not correctly perform all three operations on the matrix (transpose, change signs and add 7). Many weaker candidates produced highbred versions incorporating elements of both methods. A substantial minority introduced 9 unknowns, often treating it as an allocation problem.

8. Many candidates struggled to answer this question and those that did make a decent attempt encountered difficulties and made a succession of errors. For those candidates who made a reasonable attempt at the question, by far the most popular approach was to divide the probabilities by the value of the game. Unfortunately this then created additional difficulties for the candidate, as for player A it was then necessary to **minimise**, which a large number of candidates failed to state. Candidates also either failed to turn their inequalities into equations or added slack variables when, in this instance, they should have been subtracted. Candidates also failed to define their probabilities etc. Most candidates who tried to answer the question in this way then used the equations for player A in their simplex tableau, not realizing that they needed to change these to player B’s perspective to allow them to **maximise**. Other candidates who started in the same manner, incorrectly set up their equations (inequalities) from B’s perspective, but were then able to use these in their tableau. A minority of candidates adopted one of the other approaches and these candidates were generally more successful. Many candidates failed to mention that simplex was necessary because it was a 3 x 3 problem and it could not be reduced by dominance arguments. Those candidates who reached a correct initial tableau were generally able to manipulate this correctly, although minor errors, either arithmetic or omitting the change of base variable, did occur. Some candidates failed to state the row operations that they had used.

9. This was very poorly done by most candidates, with the exception of part (a) which was well done. In part (b) some candidates did not state which specific row/column dominated which row/column. Part (c) was certainly the most poorly handled part of the paper. Very few defined their variables and only around half the candidates remembered to make all the entries positive by adding eg 3. Very few indeed then correctly found the inequalities. Many created inequalities using the coefficients given by each row in turn rather than those given by the columns. Many candidates reversed the inequalities and very few correctly stated the objective.

10. Many candidates did not give a correct definition, with ‘sums of gains and losses equal zero’ instead of ‘sums of gains equal zero’ being popular. Part (b) was well-handled this time by most candidates. Most candidates were able to set up three equations but graphs were sometimes very poorly drawn, with common errors being lack of ruler, no scale, incorrect lines drawn and the domain extending beyond 1 or below zero. Most candidates were able to select the correct point but the other two points were both frequently chosen by candidates. The strategy or value was omitted by some. Many poor responses to part (d) were seen, many candidates did not define the probabilities or adapt the matrix to B’s perspective. Candidates frequently made errors in their constraints.

Very many candidates did not answer the question asked in part (c) and did not draw the matrix from his perspective but instead attempted to solve the game for Freddie, wasting time and gaining no marks.

11. The definitions proved tricky for many candidates, with few scoring full marks. Many referred to the ‘sum of gains and loses being zero’ in part (a) rather than the gain being equal to the loss. In part (b) many stated that the row maximin must equal the column minimax and many that the game was stable, but few stated both.

12. Most candidates found the row maximin and the column minimax but many did not make the correct link. Most candidates were able to set up three equations but many errors were seen when simplifying these. Graphs were sometimes very poorly drawn, with common errors being lack of ruler, poor scale, no scale, incorrect lines drawn and only partial domain. Of that majority who did draw the graphs correctly many were unable to find the correct intersection point, either due to choosing incorrectly or (more alarmingly) failing to solve the simultaneous equations correctly. Very many candidates did not answer the question asked in part (c) and did not draw the matrix from his perspective but instead attempted to solve the game for Freddie, wasting time and gaining no marks.

13. This was often very poorly done. In part (a) many candidates were unable to write the matrix for B, confusing signs and shape. In part (b) many candidates did not make all values positive, did not correctly define their variables, omitted slack variables and were confused whether to maximise or minimise.

14. Many candidates attempted to answer part (a) without stating row minimums or column maximums even though some then used these to show there was no stable solution in part (b). Many tried to answer parts (a) and (b) by making unsupported statements, others did not actually state their play-safe strategies. Part (c) was often well answered. Most candidates obtained the correct three equations, but a disappointingly large number failed to correctly draw a graph of them, some not even attempting to do so. Of that majority who did draw the graphs correctly, many were unable to find the correct intersection point, either due to choosing incorrectly or (more alarmingly) failing to solve the simultaneous equations correctly.

15. No Report available for this question.